International Journal of Mechanical and Industrial Technology ISSN 2348-7593 (Online)
Vol. 4, Issue 2, pp: (7-12), Month: October 2016 - March 2017, Available at: www.researchpublish.com

Stress Concentration on Rectangular Plate with Multiple Opposite Semicircular Notches Using Finite Element Analysis

Babulal K S[1], Ashenafi Adugna[2], I. Vimalkannan[3], A.Pradeep[4]

[1]Lecturer, Department of Industrial Engineering, Dire Dawa Institute of Technology, Dire Dawa University, Ethiopia,
[2]Lecturer, Head, Department of Industrial Engineering, Dire Dawa Institute of Technology, Dire Dawa University, Ethiopia
[3]Assistant Professor, Department of Mechanical Engineering, SSM Institute of Engineering & Technology, Dindigul, Tamilnadu, India
[4]Assistant Professor, Department of Mechanical Engineering, Velammal Engineering College, Chennai, India

Abstract: **In this study, stress concentration factors (SCF, K_t) of rectangular plate with multiple opposite semicircular notches are considered under uniform tensile stress so as to analyze the notch deformation due to the effect of stretching of plate. The maximum stress in the notched plate is found to determine the stress concentration factor. A plate with multiple notches under the axial load creates stress concentration near the notch and it is much larger than the average stress on the plate. Both analytical and finite element methods are used to calculate the maximum stress around the notch. SolidWorks Simulation 2014 has been employed for modelling and static analysis of linear elastic isotropic rectangular plate of size 700 x 400 x 20 mm. The uniform tensile load of pressure with a magnitude of 75 Mpa is applied on both sides of rectangular plate normal to the sides of notches with three different notch radius to plate width ratio $\left(\frac{2r}{D}\right)$ = 0.2 , 0.3 and 0.4, corresponding notch radius to center distance $\left(\frac{2r}{L}\right)$ = 0.4, 0.6 and 0.8. The result obtained on both analytical and finite element methods are compared and the percentage of error has been evaluated.**

Keywords: **Stress Concentration Factor, Notched Plate, Static Analysis, Finite Element Method.**

I. INTRODUCTION

In design aspect of any engineering structure, machinery and equipment, the stress concentration is one of the major considerations for the successful design as it can cause fracture to various mechanical components and the machine, structure may eventually get failed to function for the purpose it is supposed to be designed. The rectangular plate with holes and notches find its application in field of automobile, mechanical, aerospace and marine components. The presence of holes or notches in such components reduces the mechanical strength due to the large stress concentration near the holes or notches that causes the alleviation of fracture occurrence under service loads. Therefore, it is essential to study and analyse the state of stress around the holes and notches and also the load bearing capacity of these structures or machine components for the optimum design with safe life of component [1].The cause of highly localized or accumulation of stress near the change of cross section or clustering of stress lines at the point of discontinuity is termed as stress concentration [2].The principal cause of stress raisers like holes & notches, because concentrated stresses larger than theoretical cohesive strength will generally cause local plastic deformation and redistribution of stresses [3].There are different ways of determining the stress concentration factor in flat plates. Experimental, numerical and analytical methods are used to determine stress concentration factor [4].The study of the importance of SCF in isotropic plates is well established. Previous works on stress concentration presented a series solution for stress field around circular holes in plates with arbitrary thickness [5].A wide range of holes diameters to plate thickness with Schwarz–Christoffel transformation has been used to evaluate the stress concentration factor for an infinite plate with central triangular cut-out [6]. The numerical results based on generalized work–energy method for rectangular plates with circular cut-out and circular plates with a rectangular cut-out has been presented [7]. Ultimate strength of metallic plates with central circular cut-out under shear loading has also been investigated [8]. The stress concentration of finite composite laminates with elliptical hole and multiple elliptical holes based on classical laminated plate theory has been evaluated [9]. A mathematical analysis of the isotropic plates subjected to in plane loading is performed to calculate stress concentration factors [10].The stress concentration factor for U notches which support mixed loads is studied in which the authors use the criterion based on the deformation of the average energy density concept [11].Stress concentration in a round bar with a circular arc or V-shaped notch with bending load, tensile load and torsional load is analyzed in [12]. The comparative study of stress analysis for different types of notched plates has been explored in [13]. The present work deals with the modeling and static analysis of rectangular plate with multiple opposite semi circular notches for three different notch radius to plate width ratio in corresponding with the ratio of notch radius to center distance.

II. METHODOLOGY

A. Analytical Method:

To account for the peak in stress near a stress raiser, the stress concentration factor or theoretical stress concentration factor is defined as the ratio of the calculated peak stress to the nominal stress that would exist in the member if the distribution of stress remained uniform; that is,

$$K_t = \frac{\sigma_{max}}{\sigma_{nom}} \quad (1)$$

Nominal stress of notched member σ_{nom} is usually taken to be the axial load divided by the cross sectional area measured at the notch (i.e., Area taken remotely from notch minus area corresponding to notch)

$$\sigma_{nom} = \frac{\text{Axial Load}}{\text{Cross Sectional Area}} \quad (2)$$

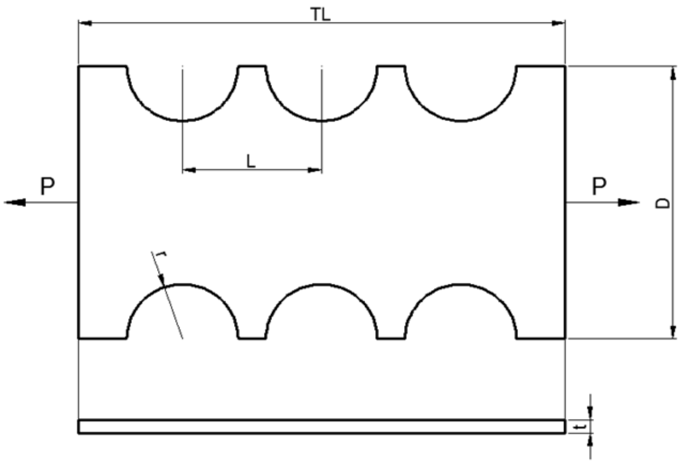

Fig.1: Rectangular plate with multiple opposite semi-circular notches

The figure 1 shows the rectangular plate of size 700 x 400 x 20 mm, with multiple opposite semi-circular notches which has been used for the analysis. It is considered three different dimensions of the notches, keeping width of the plate and center distance between the notches as constant. This plate is subjected to tensile load so as to find the stress in plate around the notches.

B. Calculation:

The stress concentration factor for the rectangular plate with multiple opposite semi-circular notches can be determined by the equation (3)

$$K_t = C_1 + C_2\left(\frac{2r}{L}\right) + C_3\left(\frac{2r}{L}\right)^2 + C_4\left(\frac{2r}{L}\right)^3, \text{ for } \left(\frac{2r}{D}\right) \leq 0.4 \text{ and } 0 \leq \left(\frac{2r}{L}\right) \leq 1.0 \quad (3)$$

Table I: Values of constant for multiple opposite semi-circular

C_1	$3.1055 - 3.4287\left(\frac{2r}{D}\right) + 0.8522\left(\frac{2r}{D}\right)^2$
C_2	$-1.4370 + 10.5053\left(\frac{2r}{D}\right) - 8.7547\left(\frac{2r}{D}\right)^2 - 19.6273\left(\frac{2r}{D}\right)^3$
C_3	$-1.6753 - 14.0851\left(\frac{2r}{D}\right) + 43.6575\left(\frac{2r}{D}\right)^2$
C_4	$1.7207 + 5.7974\left(\frac{2r}{D}\right) - 27.7463\left(\frac{2r}{D}\right)^2 + 6.0444\left(\frac{2r}{D}\right)^3$

1. ***Maximum stress for*** $\left(\frac{2r}{D} = 0.2\right)$ ***and*** $\left(\frac{2r}{L} = 0.4\right)$

$\sigma_{Max} = K_t \cdot \sigma_{nom}$ $K_t = 2.1936$

$\sigma_{Nom} = \frac{P}{t[D-2r]}$

Where,

P = Axial Tensile Force in N = Pressure x (D x t)

D = Plate Width in mm

r = Notch Radius in mm

t = Plate Thickness in mm

$\sigma_{Nom} = \frac{600000}{6400} = 93.75 \text{ N / mm}^2$

$\sigma_{Max} = 2.1936 \times 93.75 = 205.65 \text{ N / mm}^2$

2. **Maximum stress for** $\left(\frac{2r}{D} = 0.3\right)$ **and** $\left(\frac{2r}{L} = 0.6\right)$

$\sigma_{Max} = K_t \cdot \sigma_{nom}$ $K_t = 1.9251$

$\sigma_{Nom} = \frac{P}{t[D-2r]}$

$\sigma_{Nom} = \frac{600000}{5600} = 107.143 \text{ N / mm}^2$

$\sigma_{Max} = 1.9251 \times 107.143 = 206.261 \text{ N / mm}^2$

3. ***Maximum stress for*** $\left(\frac{2r}{D} = 0.4\right)$ ***and*** $\left(\frac{2r}{L} = 0.8\right)$

$\sigma_{Max} = K_t \cdot \sigma_{nom}$ $K_t = 1.7429$

$\sigma_{Nom} = \frac{P}{t[D-2r]}$

$\sigma_{Nom} = \frac{600000}{4800} = 125 \text{ N / mm}^2$

$\sigma_{Max} = 1.7429 \times 125 = 217.863 \text{ N / mm}^2$

C. Finite Element Method:

The finite element method is a numerical technique for obtaining approximate solution to a wide variety of engineering problems. Finite element program SolidWorks simulation 2014 has been used to obtain the solution of the model which is being analyzed in this study.

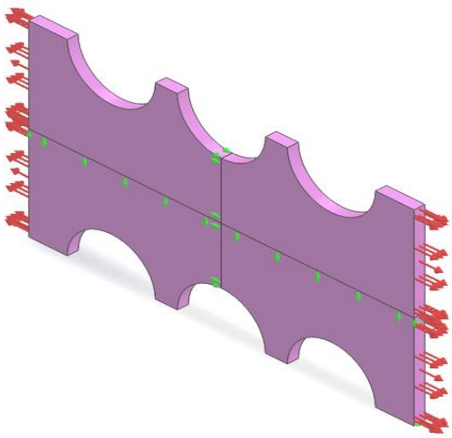

Fig.2: 3D model of rectangular plate with multiple opposite semi-circular notches under constrain and axial tensile pressure load

The figure 2 shows the 3D model of the rectangular plate with multiple opposite semi-circular notches. For the static analysis of plate, it is important to apply fixtures which prevent the out of plane rotations and free body motions there by the plate is subjected to uniform tensile load of 600000 N. Two fixtures are applied along horizontal and vertical mid-plane so as to prevent motion in the respective direction. One more fixture is applied on the vertex at the lower left corner in order to prevent motion normal to the plane. The model type of linear elastic isotropic has been selected with alloy steel of mechanical properties as shown in table II.

Table II: Mechanical properties of alloy steel

Property	**Value**	**Units**
Elastic Modulus	210000	N/mm^2
Poisson's Ratio	0.28	N/A
Shear Modulus	79000	N/mm^2
Density	7700	kg/m^3
Tensile Strength	724	N/mm^2
Yield Strength	620	N/mm^2

III. RESULT AND DISCUSSION

The result obtained from both analytical and finite element methods yield very good agreement that the variation between two methods are within the acceptable limits. The figure 3-5 shows the analysis of maximum stress experience by the rectangular plate with multiple opposite semi-circular notches of three different notch radius to plate width ratio (0.2 , 0.3 and 0.4), corresponding notch radius to center distance (0.4, 0.6 and 0.8) under the axial tensile load.

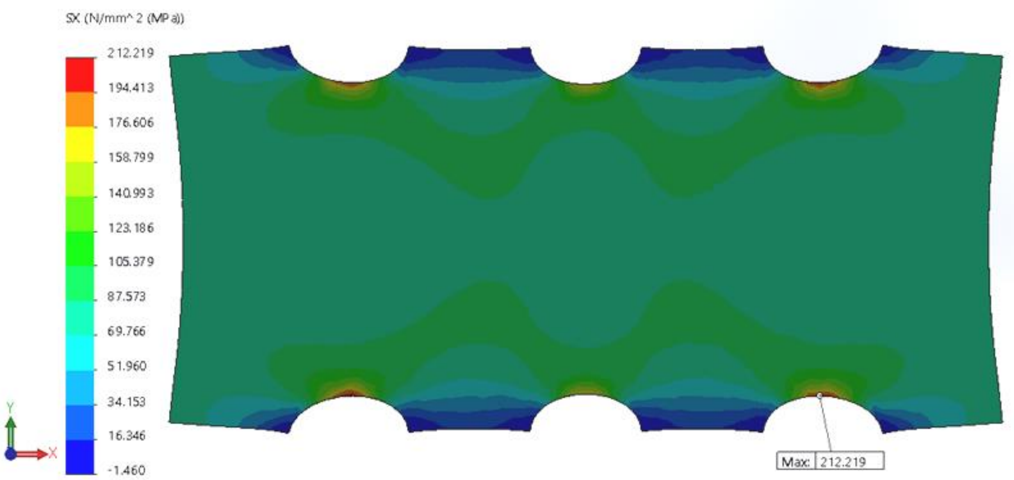

Fig.3 Effect of notch on the stress in 'x' direction for $\left(\frac{2r}{D} = 0.2\right)$ **and** $\left(\frac{2r}{L} = 0.4\right)$

International Journal of Mechanical and Industrial Technology ISSN 2348-7593 (Online)
Vol. 4, Issue 2, pp: (7-12), Month: October 2016 - March 2017, Available at: www.researchpublish.com

The maximum stress in the direction of 'x' for the rectangular plate with multiple opposite semi-circular notch radius to plate width ratio 0.2 is 212.219 Mpa which is experienced in the plate around the notch as expected shown in figure 3.

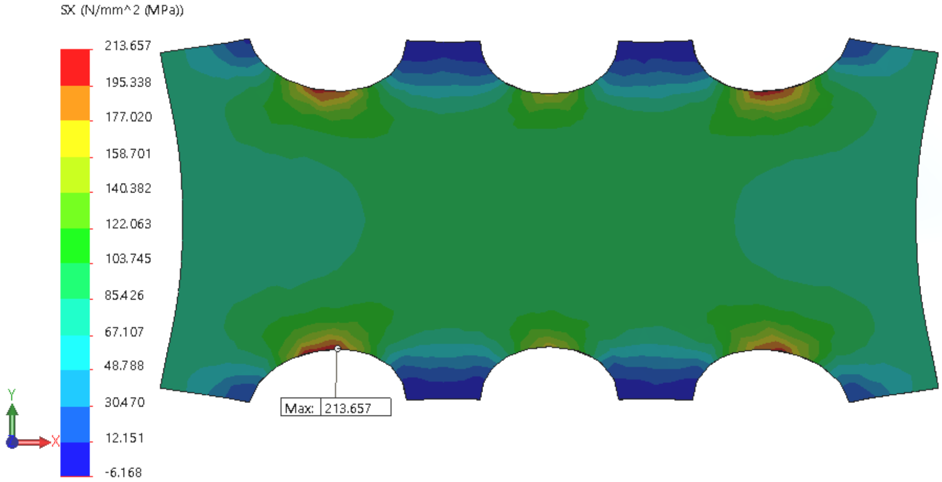

Fig.4 Effect of notch on the stress in 'x' direction for $\left(\frac{2r}{D} = 0.3\right)$ and $\left(\frac{2r}{L} = 0.6\right)$

The figure 4 shows maximum stress in the direction of 'x' for the rectangular plate with multiple opposite semi-circular notch radius to plate width ratio 0.3 is 213.657 Mpa which is experienced in the plate around the notch as expected. The magnitude of stress increases due to the increase of notch radius which reflects in the deformation of the plate.

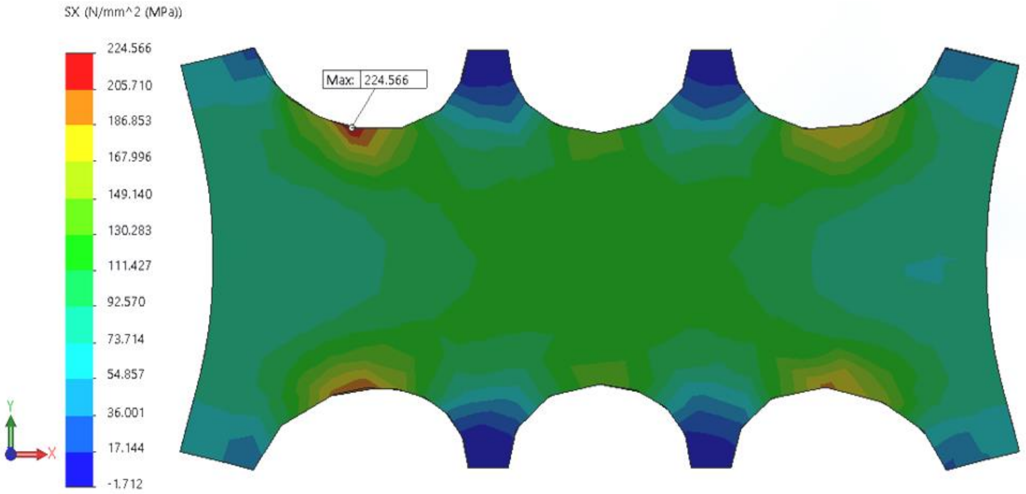

Fig.5 Effect of notch on the stress in 'x' direction for $\left(\frac{2r}{D} = 0.4\right)$ and $\left(\frac{2r}{L} = 0.8\right)$

The figure depicts the maximum stress in the direction of 'x' for the rectangular plate with multiple opposite semi-circular notch radius to plate width ratio 0.4 is 224.566 Mpa which is experienced in the plate around the notch as expected. The magnitude of stress is more than the other two cases due to increase notch radius and hence the deformation is higher as shown in figure 5.

The percentage of error between analytical and finite element method for the maximum stresses experienced by rectangular plate with multiple opposite semi-circular notch has been presented in the table III

Table III: Error percentage of σ_{Max}

S.No	$\left(\frac{2r}{D}\right)$	$\left(\frac{2r}{L}\right)$	σ_{Max} Analytical Method N/mm^2	σ_{Max} Finite Element Method N/mm^2	Error Percentage(%)
1.	0.2	0.4	205.65	212.219	3.2
2.	0.3	0.6	206.261	213.657	3.6
3.	0.4	0.8	217.836	224.566	3.1

IV. CONCLUSION

The following conclusions have been reported from the static analysis

1. The maximum stress occurs around the notches for all the cases.
2. Higher the ratio of $\left(\frac{2r}{D}\right)$ and $\left(\frac{2r}{L}\right)$**,** higher the maximum stresses in the notches.
3. The deformation of the notches also depend on the ratio of $\left(\frac{2r}{D}\right)$ and$\left(\frac{2r}{L}\right)$.
4. The observation of the error percentages between the analytical and finite element methods are 3.2 %, 3.6 % and 3.1 % for three different cases.

REFERENCES

[1] Babulal K S, I.Vimalkannan, A.Pradeep, P.Muthukumar, "SCF on isotropic rectangular plate with central circular hole using finite element analysis", International Journal of Mechanical and Production Engineering, Vol. 3, Issue-5, pp.114-117, May. 2015.

[2] M.M Leven, "Photo elastic determination of the stresses at oblique openings in plates and shells", WRC Bulletin 153, pp. 52-80, 1970.

[3] P. Stanley, B.V. Day "Stress concentrations at an oblique hole in a thick flat plate under an arbitrary in-plane biaxial loading", Journal of Strain Analysis for Engineering Design, Vol. 28, pp. 223-235, July 1993.

[4] Nagpal S, Jain N, and Sanyal S, "Stress concentration and its mitigation techniques in flat plate with singularities - A critical review", Engineering Journal, Vol. 16, Issue 1, pp. 1-16, 2012.

[5] E.S. Folias, J.J. Wang, "on the 3-dimensionl stress field around a circular holes in a plate of arbitrary thickness", Computational Mechanics, Vol.6, Issue 3, pp. 379-391, 1990.

[6] P.S. Theocaris, L. Petrou "Stress distributions and intensities at corners of equilateral triangular holes", International Journal of Fracture, Vol. 31, pp. 271-289, 1986.

[7] A.V. Singh, U.K. Paul, "Finite displacement static analysis of thin plate with an opening - A variational approach", International Journal of Solids and Structures, Vol. 40, pp. 4135-4151, 2003.

[8] Jeom Kee Paik, "Ultimate strength of perforated aluminium plates under edge shear loading", Thin-Walled Structures, Vol. 45, pp. 301-306, 2007.

[9] X. Xiwu, S. Liangxin, and F. Xuqi, "Stress concentration of finite composite laminates weakened by multiple elliptical holes," International Journal of Solids Structures, vol. 32, Issue 20, pp. 3001-3014, 1995.

[10] Pilkey, W. D. and R. E. Peterson, "Peterson's stress concentration factors", 2nd. NewYork Wiley, 1997.

[11] Gomez F. J, Elices M, Berto F, and Lazzarin P, "A generalised notch stress intensity factor for U-notched components loaded under mixed mode", Engineering Fracture Mechanics, Vol. 75, Issue 1, pp. 4819-4833, 2008.

[12] Noda N. A, and Takase Y, "Stress concentration formula useful for all notch shape in a round bar (comparison between torsion, tension and bending)", International Journal of Fatigue, Vol. 28, Issue 1, pp. 151-163, 2006.

[13] Babulal K S, I.Vimalkannan and A.Pradeep, "Comparative study of stress analysis on notched plates between analytical and finite element solutions", International Journal of Current Research and Academic Review, Vol. 3, Issue-1, pp.144-154, Jan. 2015.